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A Green–Naghdi Model in a 2D Problem of a Mode I Crack in an Isotropic Thermoelastic Plate

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Abstract—In this article, the generalized thermoelastic theory under Green and Naghdi models are used to study the thermoelastic interaction in an isotropic material containing a finite crack inside the material. The crack boundary is due to a prescribed temperature and stress distribution. Based on the Green–Naghdi type II and type III models, the formulation is applied to generalized thermoelasticity with an appropriate choice of parameters. Numerical solutions of the displacement components, temperature, and stress components are obtained using the finite element method. The results have been verified numerically and are represented graphically. Comparisons were made with expected results from Green and Naghdi model of type III and Green and Naghdi model of type II.

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1. INTRODUCTION

Two generalized thermoelasticity theories well-investigated and well-established. Replacing the classical Fourier law by postulating a new thermal conduction law, the theory of generalized thermoelasticity containing one relaxation time has been proposed by Lord and Shulman [1]. Green and Lindsay [2] established the generalized of thermoelastic theory containing two relaxation times. For the anisotropic medium, Dhaliwal and Sherief [3] extended the generalized thermoelastic theories. Entropy based on equality rather than inequality usually entropy, Green and Naghdi [4–6] established three new theories of thermoelasticity. The constitutive hypotheses for the heat flux vector in each theory are different. So they got three theories of thermoelasticity called types I, II, and III. We get the classical thermoelasticity system when the type I model is linearized. Type II model (a limiting case of type III) does not admit energy dissipation.

The strength of a material with cracks is an attracting problem in fracture and the knowledge of elastic stress fields is potentially useful for strength estimation based on the theory of brittle fracture. Several articles have appeared which treat the stress distributions in an unbounded solid due to the application of normal pressure or temperature on the faces of a circular internal fiat crack. Mathematically, the basic equations for cracking problems in piezoelasticity and magnetoelectroelasticity are identical to their analogues in pure elasticity as in Ref. [7]. Sherief and El-Maghraby [8, 9] studied mode I crack problems using the method of regularization. Prasad et al. [10] applied the method of regularization in a two dimensional thermoelastic problem of a mode I crack under Green and Naghdi type III model. Lotfy and Othman [11] studied the effect of magnetic field for a mode I crack on a two-dimensional problem under generalized thermoelastic theory. Abdel-Halim and Elfalaky [12] studied an unbounded thermoelastic solid with internal penny-shaped crack. Elfalaky and Abdel-Halim [13] investigated an unbounded thermoelastic space containing a mode I crack.

The analytical solution of the basic equations of the generalized thermoelastic theory for a coupled and linear/nonlinear system exists only for very special and simple initial and boundary issues. Therefore one can chose the finite element method. Three steps have been involved to apply the finite element method. The first step is to take the overall behavior of the variables so as to satisfy the differential equations given unknown field. The second step is temporal integration. The temporal derivatives of the unknown variables must be determined by the previous results. In the third step, the solutions of equations resulting from the first and second steps will obtained by the finite element algorithm as in Ref. [14].

The present paper investigates a GN model in a two dimensional problem of a mode I crack in a thermoelastic medium using the finite element method. The results have been verified numerically and represented graphically.

2. BASIC EQUATION

For a homogenous, isotropic, and linear thermoelasticity, the basic equations can be written in the form [10]

$$\mu u_{i,jj} + (\lambda+\mu)u_{j,ij} - \gamma T_{,i} = \rho \ddot{u}_i,\ i, j = 1, 2, 3, \quad (1)$$

where λ, μ are elastic parameters, ρ is the mass density, u_i are the components of displacement, T is the change in temperature of a particle of material, $\gamma = (3\lambda+2\mu)\alpha_t$, α_t is the linear thermal expansion coefficient and t is the time.

The form of heat equation can be given by

$$K^* T_{,ii} + nK\dot{T}_{,ii} = \rho c_e \ddot{T} + \gamma T_o \ddot{u}_{i,i},\ i, j = 1, 2, 3, \quad (2)$$

where K is the thermal conductivity, c_e is the specific heat at constant strain, T_o is the reference uniform temperature, K^* is the material constant characteristic of the theory, $n = 1$ refers to the theory of Green and Naghdi of type III (with energy dissipation) while $n = 0$ refers to the theory of Green and Naghdi of type II (without energy dissipation). The constitutive equations have the form

$$\sigma_{ij} = (\lambda u_{i,i} - \gamma(T - T_o))\delta_{ij} + \mu(u_{j,i} + u_{i,j}), \quad (3)$$
$$i, j = 1, 2, 3,$$

where δ_{ij} is the Kronecker symbol, and σ_{ij} are the stress components.

3. FORMULATION OF THE PROBLEM

An infinite space $-\infty < x < \infty$, $-\infty < y < \infty$ was considered in this problem with a crack on the x axis, $|x| \le a$, $y = 0$. The surface of the cracks is subjected to a prescribed temperature and to the normal stresses. The displacement components u_i are $(u(x, y, t), v(x, y, t), 0)$. In this case, the governing equations have the following form [10]:

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 v}{\partial x \partial y} + \mu\frac{\partial^2 u}{\partial y^2} - \gamma\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \quad (4)$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x \partial y} + \mu\frac{\partial^2 v}{\partial x^2} - \gamma\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2}, \quad (5)$$

$$\left(K^* + nK\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \frac{\partial^2}{\partial t^2}\left(\rho c_e T + \gamma T_o\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right), \quad (6)$$

$$\sigma_{xx} = (\lambda+2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y} - \gamma(T - T_o), \quad (7)$$

$$\sigma_{yy} = (\lambda+2\mu)\frac{\partial v}{\partial y} + \lambda\frac{\partial u}{\partial x} - \gamma(T - T_o), \quad (8)$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \quad (9)$$

It should take the nondimensional form for the previous equations. Thus, the nondimensional parameters are given by

$$T' = \frac{T - T_o}{T_o},\ (v, u', x', y') = \frac{c}{\chi}(v, u, x, y),$$
$$(\sigma'_{xx}, \sigma'_{xy}, \sigma'_{yy}) = \frac{1}{\mu}(\sigma_{xx}, \sigma_{xy}, \sigma_{yy}),\ t' = \frac{c^2 t}{\chi}, \quad (10)$$

where $\chi = K/(\rho c_e)$ and $c^2 = (\lambda+2\mu)/\rho$.

In terms of the dimensionless quantities (10), after neglecting the primes, the above equations can be reduce to

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + (\beta^2 - 1)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}\right) = \alpha\frac{\partial T}{\partial x} + \beta^2\frac{\partial^2 u}{\partial t^2}, \quad (11)$$

$$\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} + (\beta^2 - 1)\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) = \alpha\frac{\partial T}{\partial y} + \beta^2\frac{\partial^2 v}{\partial t^2}, \quad (12)$$

$$\left(n\frac{\partial}{\partial t} + \varepsilon_1\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \frac{\partial^2}{\partial t^2}\left(T + \varepsilon_2\left(\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right)\right), \quad (13)$$